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Frequency and development of slugs in a horizontal pipe at large liquid flows

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Abstract

The generation of slugs was studied for air–water flow in horizontal 0.0763 m and 0.095 m pipes. The emphasis was on high liquid rates ($u_{LS} \geqslant 0.5$ m/s) for which slugs are formed close to the entry and the time intervals between slugs are stochastic. A "fully developed" slug flow is defined as consisting of slugs with different sizes interspersed in a stratified flow with a height slightly larger than the height, h_0, needed for a slug to be stable. Properties of this "fully developed" pattern are discussed. A correlation for the frequency of slugging is suggested, which describes our data as well as the data from other laboratories for a wide range of conditions. The possibility is explored that there is a further increase of slug length beyond the "fully developed" condition because slugs slowly overtake one another.

The evolution of slugs from a highly disturbed stratified flow at the inlet was studied by measuring the holdup at a number of locations along a 20 m length of 0.0763 m pipe. At superficial gas velocities less than 3 m/s incipient slugs form by waves touching the top of the pipe very close to the entry. At $U_{SG} > 3$–4 m/s incipient slugs form by wave coalescence farther downstream. These incipient slugs grow as long as the height of the stratified flow in front of them is greater than h_0. When equal to h_0, the slug can propagate downstream or decay if it has not reached a stable length. Decaying slugs form large amplitude roll waves which propagate downstream at a lower velocity than slugs. Slugs that overtake these waves increase in size. These results can be used in developing a stochastic model for the evolution.

Keywords: Slug flow; Frequency; Length distribution; High liquid flow; Air–water

1. Introduction

Slugging is a commonly observed pattern in horizontal and near horizontal gas–liquid flows. It is characterized by the intermittent appearance of aerated masses of liquid that fill the whole cross-section of a pipe and travel downstream at a large velocity. A number of investigators have used a model for predicting pressure drop and liquid holdup that pictures slug flow as consisting of a sequence of aerated turbulent units that translate at a constant velocity in a stratified liquid layer (Fan et al., 1993b; Dukler and Hubbard, 1975). A critical

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issue in developing these models is the prediction of slug frequency and the distribution of slug lengths. This paper describes work which is a continuation of studies in this laboratory that have examined the formation of slugs so as to provide a basis for developing a theory for the distributions of slug lengths and the time intervals between slugs. An important finding is that there are a number of mechanisms (Woods and Hanratty, 1996, 1999; Woods et al., 2000; Simmons and Hanratty, 2001; Solemani and Hanratty, 2003; Hurlburt and Hanratty, 2002; Lin and Hanratty, 1986a,b). The focus of this study is the formation of slugs for an air–water flow in a horizontal pipe at high enough liquid velocities that they evolve by a stochastic process from waves which are created close to the entry (Woods and Hanratty, 1999; Hanratty and Woods, 2001).

Fan et al. (1993a) studied the transition from a stratified to a slugging pattern for air–water flow at atmospheric pressure in a 0.095 m pipeline. The entrance was a box with dimensions of $0.45 \times 0.43 \times 0.37$ m that is connected to the pipeline by a converging pipe with an angle of 5°. The air entered at the top of the box, and the liquid at the bottom. A pool was formed in the box. This mixing section created less disturbances than a simple tee because of the liquid holdup in the box and because there was a favorable pressure gradient in the converging section.

At low gas velocities slugs are initiated on a stratified flow from waves with a length of about 0.085 m, which are generated by a Jeffreys (1925) mechanism. These waves grow in height and eventually double in wave length by a nonlinear process. Depending on the height of the liquid, the growth can lead to a breaking wave or to a wave that fills the whole pipe cross-section (Fan et al., 1993a).

At high gas velocities (that is, greater than 4 m/s), capillary-gravity waves with a wide range of lengths are generated by a Kelvin–Helmholtz mechanism (Andritsos and Hanratty, 1987). These rapidly evolve into long waves that could not be initiated by a linear mechanism. If the height of the wavy stratified flow is large enough to sustain stable slugs, $h > h_0$, initiation occurs through wave coalescence (Lin and Hanratty, 1987; Fan et al., 1993a; Andritsos et al., 1989).

In a paper by Woods and Hanratty (1999), to be referenced as WH, results are presented on the formation of slugs for an air–water flow in a 0.0763 m pipe for flow conditions that are larger than the critical. The liquid and gas were combined at the beginning of the pipeline in a specially designed tee section which contained an insert to separate the two phases before they entered the pipe. Several tees with different inserts were used to introduce the liquid at a height close to the equilibrium value, h_e. A critical orifice was installed just upstream of the gas inlet to prevent the triggering of slugging due to a sudden increase in the gas flow caused by the discharge of slugs from the pipeline.

For small gas and liquid flows, slugs form at large distances downstream of the entry, say greater than 40 pipe diameters. The Froude number (defined with the liquid height and velocity) characterizing the stratified flow at the inlet can be less than unity. WH showed that under these circumstances the depleted liquid layer behind a slug is replaced by a depression wave that moves backward and is reflected at the inlet to locations where unstable waves are formed. The time required for this to occur dictates the time interval between slugs; the process is regular.

At high gas velocities and low liquid velocities, WH found that, for supercritical conditions, slugs form through a stochastic process. The Froude number is greater than unity at the inlet, so the depleted liquid is replaced by a hydraulic jump. They suggested that the probability of forming a new slug at a given time depends on the length of the unstable stratified flow between the entry and the front of the jump.

At high liquid flows ($U_{LS} >$ ca. 0.2 m/s) more than one slug can exist in the short pipes used in the laboratory. Slugs are formed close to the inlet by stochastic processes that are not completely understood. Large amplitude waves evolve through growth or through coalescence of smaller waves to form disturbances which fill the whole pipe cross-section. These can lengthen and persist as slugs to the pipe outlet or they can decay and carry liquid down the pipe as large amplitude roll waves (Woods and Hanratty, 1999, pp. 1204–1205; Hanratty and Woods, 2001).

These flows are the focus of this study, which has several aspects: (1) We present results from studies of the slug frequency at $u_{LS} \geqslant 0.5$ m/s in two systems (described above) with quite different entry conditions. One study in a 0.095 m pipe considers unpublished results from experiments by Fan et al. (1993a,b). The other was done in a 0.0763 m pipe at large liquid flows by Woods and Hanratty (1999). A general correlation for slugging frequency at large liquid flows, which considers studies from several laboratories, is presented. (2) Previous work has indicated that a stratified flow must exceed a critical height, h_0, for slugs to appear. We show that the signature of an incipient slug is the sudden drop of the liquid level behind a disturbance. Further development of this incipient slug results in the appearance of a stratified flow with $h \cong h_0$ behind the slug. (3) We use our measurements and studies from other laboratories to define a "fully developed" slug pattern which consists of slugs, with a wide range of lengths and time intervals, moving over a stratified layer of height h_0. (4) The finding that slugs in petroleum fields have lengths which are an order of magnitude larger than what is observed in laboratories has lead to the suggestion that further changes beyond the "fully developed" condition occur through mergers of slugs. This possibility is discussed. (5) At high liquid rates, the frequency of interfacial disturbances undergoes an enormous change from the pipe entrance to the outlet. Physical processes responsible for this change are identified, in the 0.0763 m pipe, through photographic studies and measurements with parallel wire conductance probes that span the whole pipe diameter and are located along the length of the pipe. The methods employed are described in WH and in Woods (1998). (6) We suggest that a critical theoretical issue in obtaining a physical understanding of the "fully developed" slug pattern at high liquid rates is the construction of a stochastic computational scheme which defines the development process. Elements of such an approach are identified.

WH have shown that theories by Taitel and Dukler (1997) and by Troconi (1990) for the frequency of regular slugging are not applicable to the results discussed in this paper, so they are not explored.

2. Theoretical considerations

2.1. Balance equations

Simple, one-dimensional balance equations (used by many previous authors) are employed in this section to define a critical height of a stratified flow, h_0, below which stable slugs cannot be generated. We show that evolving slugs, on a layer with $h > h_0$, have a depleted layer in their rear, which assumes a height, h_0, as a 'fully developed' pattern is approached. A simplified equation relating the slug frequency to the slug length is derived from conservation of mass considerations for a "fully developed" pattern in Section 2.4. A discussion of stochastic models is presented in Section 2.5.

Consider the simplified version of a slug in Fig. 1. Station 1 represents the stratified layer in front of the slug. Stations 2 and 3 are, respectively, located in the body of the slug and in the stratified flow behind the slug. The simplification is made that bubbles are uniformly distributed in the slug. The front has a velocity C_F and the back has a velocity C_B.

As a slug propagates downstream it picks up liquid at a rate given as

$$Q_{IN} = (C_F - u_1)A_{L1} \tag{1}$$

where u_1 is the velocity of the liquid in the stratified flow in front of the slug and A_{L1} is its area.

For long enough slugs, Q_{out} and C_B are independent, or weakly dependent, on slug length. Growth, or decay, then depends on the rate at which the slug accumulates liquid, Q_{in}. A material balance for a control volume which is attached to the back of the slug and moving with it gives

$$Q_{out} = (C_B - u_2)(A)(1 - \alpha) \tag{2}$$

where α is the void fraction in the slug. Since the flow is considered to be incompressible, the volumetric flow at any cross section is given by $U_M A$, where U_M is equal to the sum of the superficial gas and liquid velocities, $U_M = U_{SG} + U_{SL}$. Therefore, the velocity in the body of the slug, u_2, can be calculated as

$$u_2 = \frac{U_M}{s\alpha + (1 - \alpha)} \tag{3}$$

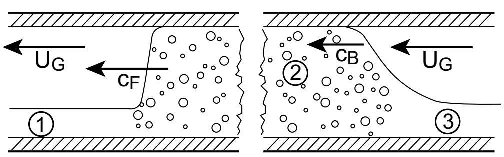

Fig. 1. Sketch of a slug.

where s is the velocity ratio of the bubbles in the slug. The void fraction, α, is obtained from experiments. The shedding rate, Q_{out}, has been measured directly or calculated with (2) by using a correlation for C_B (Woods and Hanratty, 1996).

2.2. Definition of u_0 and A_{L0} for a stationary slug

For a stationary slug, $Q_{in} = Q_{out}$ and $C_F = C_B$. The conditions prevailing in the front of a stationary slug (u_0, A_{L0}) can, therefore, be calculated from Q_{out} by using Eq. (1) and a model for the stratified flow, in front of the slug, that includes momentum considerations. The height at the centerline of the stratified flow, h_0, is calculated from A_{L0} by using geometric relations that assume the interface is horizontal.

For A_{L1}, h_1 greater than A_{L0}, h_0 the slug grows. For A_{L1}, h_1 less than A_{L0}, h_0, the slug decays. This notion of a necessary condition for the existence of slugs has been used in previous papers to explain the critical conditions for the existence of slugging. The method for calculating h_0 is outlined in Woods and Hanratty (1996). The velocity, u_0, for a given h_0 can be calculated if the volumetric flow in the film or the interfacial stress on the film is known. Usually u_0 is small compared to C_F so it can be neglected.

2.3. Conditions on the stratified flow behind a growing or decaying slug

The liquid holdup changes very rapidly in the space just behind a slug and then the decreases more slowly to form a long tail. In a developing slug the shape of the tail could be changing with time. Heights less than h_0 are often realized. This behavior has been described by Jepson (Jepson, 1987; Fan et al., 1992; Ruder et al., 1989) as resembling the flow field resulting from the breakdown of a dam, analyzed by Stoker (1957).

Eventually, however, the tail loses its identity by being overtaken by large amplitude waves and develops into a steady stratified flow for which interfacial stresses are more important than gravity in determining its height.

When a steady stratified flow appears behind the slug, the shedding rate can be described as

$$Q_{out} = (C_B - u_3)A_{L3} \tag{4}$$

where A_{L3}, u_3 are the area and velocity of the stratified flow that eventually develops behind the slug. From (2) and the recognition that $C_B = C_F$, $Q_{in} = Q_{out}$ for a stationary slug,

$$Q_{out} = (C_B - u_0)A_{L0} \tag{5}$$

By comparing (4) and (5) we see that the conditions in the stratified flow behind a fully developed tail are $u_3 = u_0$, $h_3 = h_0$.

When this fully developed tail is realized the liquid in front of the slug could have a height, h_1, which is greater or less than h_0. In the former situation the slug will grow; in the latter it will decay. For a flow with stationary slugs the conditions in the front and back of the slugs should be the same. Thus, a "fully developed" slug flow is pictured as slugs with different lengths moving over a stratified flow with height and velocity equal to h_0 and u_0.

2.4. Slug frequency

The slug frequency, f_S, can be related to the slug length, L_S, for a fully developed flow consisting of slugs embedded in a stratified layer with height h_0 and velocity u_0. An intermittency, I, is defined as the fraction of the time a slug is observed by a stationary observer,

$$I = \frac{f_S L_S}{C} \tag{6}$$

The intermittency can be very roughly approximated as

$$I \sim \frac{U_{SL}}{U_{SL} + U_{SG}} \tag{7}$$

Furthermore, a number of studies (for example, Woods and Hanratty, 1996) show that

$$C = 1.2(U_{SG} + U_{SL}) \tag{8}$$

If (7) and (8) are substituted into (6), one obtains a first approximation for the relation between f_S and L_S under "fully developed" conditions.

$$\frac{f_S D}{U_{SL}} = 1.2\left(\frac{L_S}{D}\right)^{-1} \tag{9}$$

If the average value of L_S/D under the "fully developed" condition is a constant, one obtains

$$\frac{f_S D}{U_{SL}} = \text{constant} \tag{10}$$

In addition to the use of a highly simplified model of slug flow, Eq. (9) ignores the liquid carried in the stratified flow between the slugs and the gas carried by the slugs. A derivation which takes these factors into account is given by Woods (1998).

2.5. Stochastic models

The critical issue in establishing a fundamental understanding of slugging at large liquid flows is the formulation of a stochastic model for how the "fully developed" region evolves. Very little has been done on this problem, so only a brief discussion is appropriate.

Bernicott and Drouffe (1991) and WH developed stochastic models whose prerequisite is a probability function for a slug to appear at a given location, x, at a given time. This section will only focus on a modification of the WH approach, presented in a thesis by Woods (1998). As mentioned earlier, the generation of slugs at low liquid rates can occur far enough downstream that inlet conditions are not a factor. In order to capture the behavior under these circumstances WH considered low liquid flows for which slugs are formed at least 40 pipe diameters downstream. At low gas and liquid velocities, the Froude number in the stratified layer entering the pipe was less than unity and slugs were formed by the deterministic process outlined in Section 1. At high gas velocities and low liquid flows, the Froude number was greater than unity and the slugs were generated by a stochastic process, such as described below:

The height of the stratified layer entering the pipe is close to the equilibrium value for a stratified flow for specified gas and liquid rates, h_e. A development pipe length, L_D, was needed for the formation of unstable waves which have the possibility of coalescing to form a slug. Thus slugs appear within the region where unstable waves exist, $L > L_D$. The formation of a slug at $t = t_n$ and at $L_F > L_D$ is accompanied by a depletion of liquid behind the slug, where the height is assumed to be h_0. Because $Fr > 1$, a bore is formed where the level of the stratified flow changes from h_e to h_0. The bore moves downstream with a velocity C_{jump} and is supplied by liquid entering the pipe. The slug has a larger velocity than the bore so the depleted layer increase in length with increasing time. The length of the stratified layer that begins at the entry also increases with time, but at a smaller rate.

$$L(t) = L_{Fn} + C_{jump}(t - t_n) \tag{11}$$

where L_{Fn} is the location where the slug was found at t_n.

WH postulated that the probability of forming a slug somewhere in the pipe at $t > t_n$ increases linearly with the length of the unstable region, L_U,

$$\begin{aligned} L_U(t) &= L(t) - L_D \\ P(t)\mathrm{d}t &= \Delta t N L_U(t) \end{aligned} \tag{12}$$

This assumption results in a higher probability of forming longer slugs, that is, to a long tail in the probability function, such as found in a log-normal distribution.

Like Bernicott and Drouffe (1991), Woods (1998) assumed that it is equally probable for the slug to appear anywhere in the unstable region and that there is a minimum slug length, (L_S) min, below which the slug is unstable. A computational scheme used to implement the theory is given by Woods (1998).

The simulation is started by beginning with an unstable wave pattern of length $L_U(t)$. A random number, R_1, between 0 and 1 is generated. If R_1 is less than the right side of (12) a slug has formed; else, the time is incremented by another Δt and L_U is increased by the distance the bore has moved during Δt. Another random number is generated and the process is repeated until R_1 is less than $\Delta t N L_U(t)$. At this point a second random number determines where, along L_U, the new slug appears. The slug contains a volume, $A_S(L_U - L_F)$ where A_S is the area of the unstable stratified flow and $L_U(t) - L_F$ is the length of the stratified flow downstream of the locus, L_D, at which the slug is formed. The new slug is assumed to leave behind a layer with an area, A_0. The slug fills the whole cross-section of the pipe and is aerated. Thus, the initial length of the slug is

$$(L_S)\text{initial}[A(1-\alpha) - A_0] = (A_S - A_0)[L_U(t) - L_F] \tag{13}$$

where α is the void fraction in a slug. If (L_S) initial is below (L_S) min, the slug is unstable and decays.

If the slug decays the volume on left side of (13) is added to the stratified flow downstream. Subsequent slugs can increase their length by adding the liquid deposited by decayed slugs. Consequently, the slug length L_S in a fully developed flow, which initially had a length of (L_S) initial, is given by

$$L_S = (L_S^i)\text{initial} + \sum_{i=1}^{i=j-1} (L_S^i)\text{initial} \tag{14}$$

The first term on the right side is greater than (L_S) min. This is the jth disturbance since the last slug formation which resulted in a stable slug. The summation on the right side represents the slug lengths of the $j-1$ disturbances which failed to form a stable slug since the last slug formed.

The calculations of Woods and Hanratty (1999) for low U_{SL} and high U_{SG} did not use the concept of a minimum slug length. The calculated probability distribution function representing the time intervals between slugs showed a skewness that is roughly the same as is found in experiments. The constant N appearing in (12) was found to vary linearly with U_{SL} and to be weakly dependent on u_{SG}. This is consistent with the expectation that N should scale in the same way as the mean frequency of slugging.

Woods (1998) also analyzed the situation of high U_{SL} considered in this paper. The situation for which slugs form right at the entry was considered, so that L_D in Eq. (12) was set equal to zero. The dimensionless minimum slug length $(L_S)_{min}/D$ was selected to be 4.

A comparison between predicted and measured probability functions are presented in Fig. 2 for $U_{SG} = 1.2$ m/s, $U_{SL} = 1.2$ m/s and for $U_{SG} = 2.4$ m/s, $U_{SL} = 1.2$ m/s. These are for situations in which the flow is "fully developed" or close to being "fully developed" (see Fig. 6). Rough agreement between calculations and experiments is obtained. Both show skewed distributions. However, the model predicts the existence of slug lengths larger than observed in the experiments.

3. Measurement of the frequency of slugging

Slug frequency was measured with a piezoelectric pressure transducer mounted flush with the wall close to the pipe exit. A characteristic pressure pulse reveals the passage of a slug, as described by Lin and Hanratty (1986a,b, 1987), Fan et al. (1993a,b) and WH. Fig. 12 of the paper by Woods and Hanratty (1999) shows such a pressure trace for $U_{SG} = 5.5$ m/s. Fig. 6 of the same paper gives a trace of the time variations of the liquid holdups (obtained with conductance probes) and the time variation of pressure at the outlet for $U_{SG} = 5.5$ m/s. These results clearly show the advantage of using pressure traces at high gas velocities where it is difficult to differentiate between large amplitude waves and highly aerated slugs with a conductance probe.

Measurements of slugging frequency by Fan in a 0.095 m pipe are given in Fig. 3. These show an approximately linear increase with U_{SL}. For a given U_{SL} a minimum in the frequency is found at approximately $U_{SG} = 4$ m/s, as has been observed by other investigators. Bishop (1990) has noted that the minimum is at a gas velocity where irregular waves generated by a Kelvin–Helmhotlz mechanism appear (Andritsos and Hanratty, 1987).

Measurements of slug frequency by WH were made in a 0.0763 m pipe that used a tee entry. Results for $U_{SL} > 0.4$ m/s are plotted in Fig. 4. They are similar to those shown in Fig. 3 in that the frequency is proportional to U_{SL} and a minimum is observed at $U_{SG} \cong 4$ m/s.

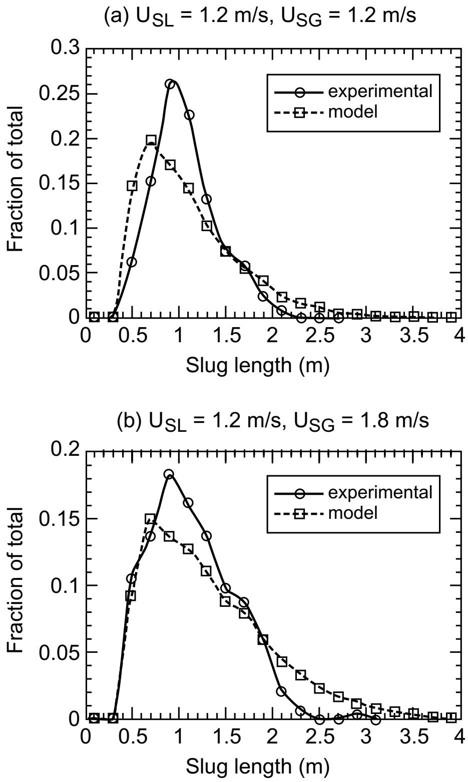

Fig. 2. Comparison of calculated and measured distributions of slug lengths.

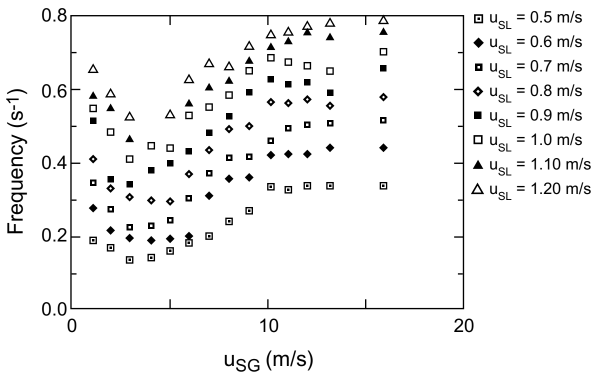

Fig. 3. Measurements of slug frequency in a 0.095 m pipe.

Fig. 5 compares measurements in pipes with diameters of 0.019 m (Gregory and Scott, 1969), 0.042 m (Heywood and Richardson, 1979), 0.0763 m (WH), 0.095 m (Fan) and 0.15 m (Crowley et al., 1986). The ordinate is $f_S D/U_{SL}$ and the abscissa is $U_{SL}/(U_{SL}+U_{SG})$. It is noted that this plot captures, approximately, the effects of liquid flow and of pipe diameter and that $f_S D/U_{SL} = 0.05$ is a rough approximation of the data.

One of the goals of this research was to discover whether entry effects are having a strong impact on the generation process. This does not appear to be the case since similar results were obtained with our measurements in 0.0763 m and 0.095 m pipes, even though their entry designs were quite different.

Measurements for $U_{SL} < 0.40$ m/s reported by WH are qualitatively different from Figs. 3 and 4 in that a minimum frequency is not observed. That is, the frequency is observed to increase with increasing U_{SG} for

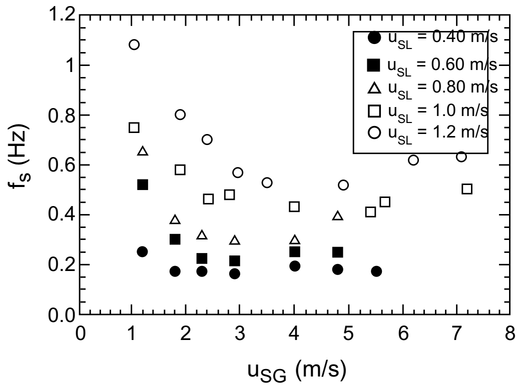

Fig. 4. Measurements of slug frequency in a 0.0763 m pipe.

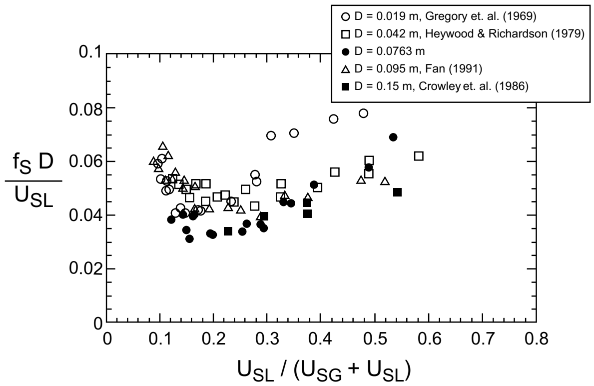

Fig. 5. Measurements of slug frequency for different pipe diameters.

$U_{SG} < 4$ m/s. The trend of the data at higher U_{SG} is not well established but there is a suggestion that the frequency of slugging is decreasing with increasing gas velocity in this region. The most striking difference is that the slugging is regular at low gas and liquid velocities; that is, the time interval between successive slugs does not vary. At $U_{SG} > 4$ m/s the process is stochastic both at high and low liquid velocities.

4. "Fully developed" slug patterns

For a given gas velocity and very small liquid velocity slugs will not be observed in the 0.0763 m pipe. As the liquid flow increases the slugs will first appear close to the outlet of the pipe. With small increases in liquid flow the location at which slugs appear is much closer to the inlet (see Fig. 23 of WH) and more than one slug exists in the pipe at a given time.

Fig. 6 considers this situation. It shows measurements of h/D close to the outlet for $U_{SL} = 1.2$ m/s and three gas velocities. As suggested in Section 2 the height of the liquid in the stratified flow behind a slug is observed to be approximately equal to h_0.

The horizontal lines, which corresponds roughly to the height of the stratified layer behind the slugs, provide measurements of h_0. These should correspond to the critical height of a stratified flow needed to sustain a slug. Calculations h_0/D that use (1)–(3) and the condition for a neutrally stable slug, $Q_{out} = Q_{in}$, are presented in Fig. 14 of the paper by Woods and Hanratty (1996). These give $h_0/D = 0.25$–0.20 for $U_{SG} = 3$–13 m/s. It is noted that the measurements for "fully developed" slug flow in Fig. 6c give a value of $h_0/D = 0.26$. Both calculations of the critical height and measurements such as shown in Fig. 6 indicate sharp increases in h_0/D for $U_{SG} < 1.2$ m/s.

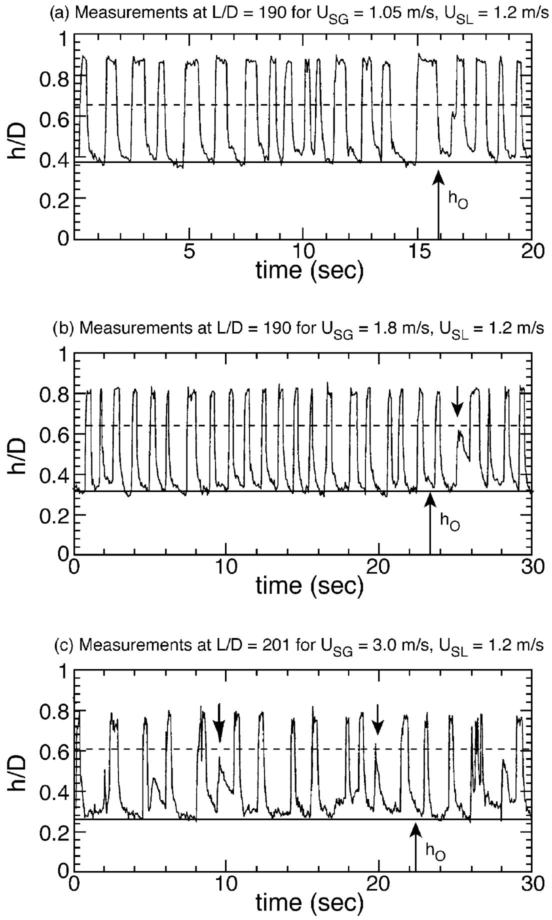

Fig. 6. Holdup measurements at $U_{SL} = 1.2$ m/s, $D = 0.0763$ m.

If the stratified flow in front of a slug has a height greater than h_0 the slug will grow at the expense of this layer. One would, therefore, expect that the slug flow eventually reaches a "fully developed" condition for which slugs move over a stratified flow with a height approximately equal to h_0. This notion is supported in Fig. 6.

The h/D measured with the conductance probes does not reach a value of unity when a slug passes because of the presence of gas bubbles. An increase of aeration with increasing gas velocity is thus indicated in Fig. 6. Values of α calculated from the peaks in h/D measurements are given in Woods and Hanratty (1996). Because of aeration and the coarse time resolution of measurements in Fig. 6 the shapes of the slugs are not accurately captured. Measurements for individual slugs are presented by Woods and Hanratty (1996). These show a sharp decrease behind the slug and a long tail over a distance of about $35D$.

The arrows in Fig. 6 indicate large amplitude waves which are moving at a lower velocity than the slugs. These are overtaken by a slug, thus causing an increase in the mass of liquid in the slug. The results in Fig. 6 indicate that fully developed slugs, as described above, are observed at the end of the pipe for $U_{SG} = 1.05$ m/s and for $U_{SG} = 1.8$ m/s. At larger U_{SG}, the frequency of slugging is not changing by a large amount with distance downstream. However, the sizes of the slugs increase by incorporating liquid contained in large amplitude waves. In a longer pipe a "fully developed" condition would occur at $U_{SG} > 1.8$ m/s after all of the large amplitude waves have been consumed. Fig. 7 presents measurements of the cumulative length distribution function. It is noted that these distributions show long tails and suggest a cut-off of about 0.4 m. The function is still changing with pipe lengths greater than $L/D = 200$ for $U_{SG} = 2.4$ m/s and $U_{SG} = 3$ m/s.

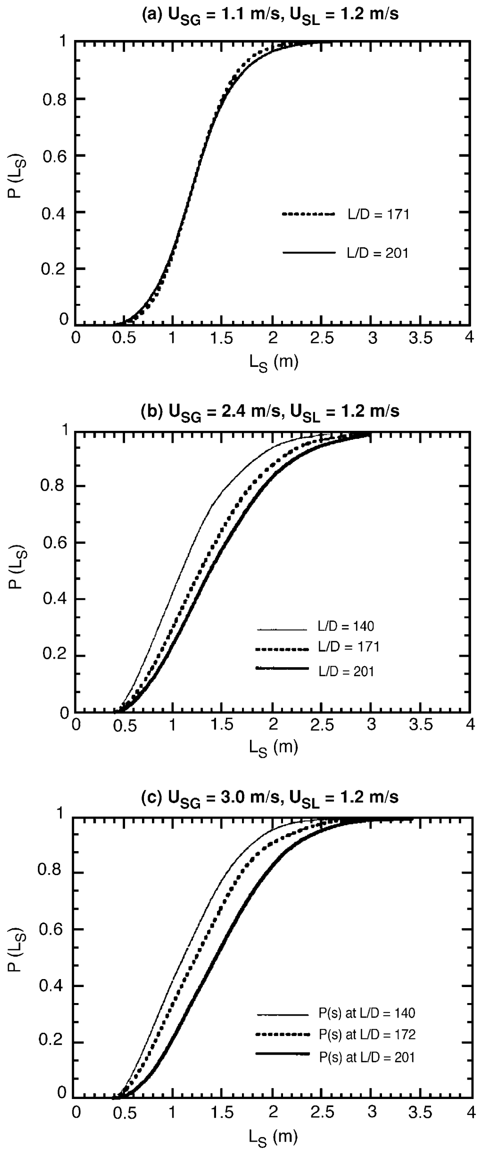

Fig. 7. Cumulative probability distributions for slug lengths, $D = 0.0763$ m.

Scott et al. (1987) have suggested two stages of slug growth from field experiments in very long petroleum pipelines at conditions close to what is needed to initiate slugging. The first involves the accumulation of waves by the slugs and would culminate as the "fully developed" pattern discussed above. The second involves the coalescence of and possibly the decay of slugs. It would have smaller growth rates than the first and might not be observed in the short pipes used in laboratories. This method of slug growth has been discussed further by Barnea and Taitel (1993).

Several mechanisms for the growth of slugs in this postulated second stage have been discussed. The most popular focuses on differences in the velocities of the gas bubbles behind two successive slugs with different lengths. This approach presumes that the bubble velocity behind a slug increases with decreasing slug length.

Measurements of the velocity of the back end of a slug, C_B, have been made in the 0.0763 m pipe by using two pairs of conductance probes located close to one another near the outlet of the pipe. The slug length is obtained from a measurement of the time needed for the slug to pass one of the probe pairs. These measurements are shown in Fig. 8. It is noted that C_B can have a small but significant range of velocities for a given

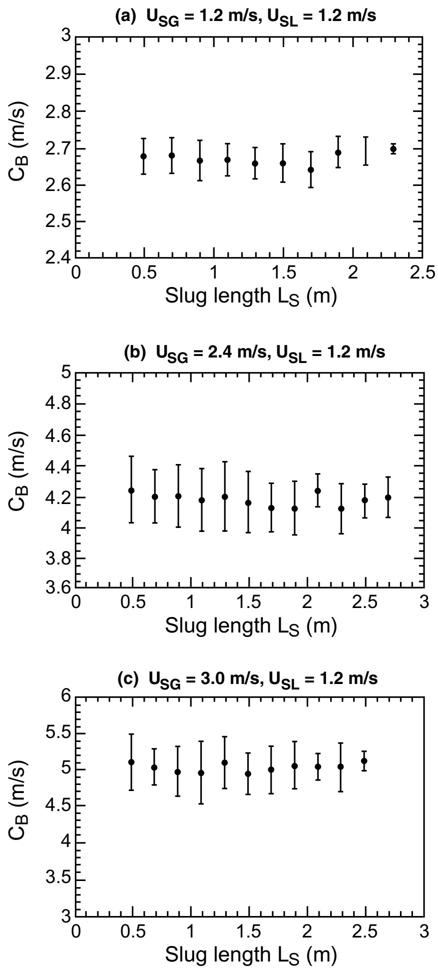

Fig. 8. Effect of slug length on the velocity of the bubble behind a slug, $D = 0.0763$ m.

U_{SL} and U_{SG}. This indicates that adjacent slugs can have different velocities, thus offering the possibility for one slug to overtake another in a sufficient time, as suggested by Scott et al. (1987). However, it is noted from Fig. 8 that C_B is independent of slug length. This seems to overrule the mechanism outlined above for the coalescence of slugs. Measurements of C_B were not made for $L_S < 6.5$ m (or $L_S/D < 6.6$). Therefore, the existence of a critical length because of an increase of C_B with decreasing slug length is not ruled out by these results.

The holdup tracings shown in Fig. 6 could be misleading in that spatial changes in the flow direction are compressed. Slug velocities are given by (6). Thus one second on the abscissa of Fig. 6b represents 3.6 m or 47 pipe diameters. That is, the lengths of the slugs are compressed in Fig. 6. For example, the smallest slugs in 6b, which take about 0.2 s to pass a conductance probe, are approximately 0.72 m or 9.5 pipe diameters in length.

Measurements of the average slug length and the distribution function (Fig. 7) at the outlet were determined by setting an h/D at which to measure the time interval for the slug passage. This trigger is selected so that it gives the correct frequency. Thus, $h/D = 0.6$ does a good job for the holdup tracings in Fig. 6. For larger gas velocities the measurements are less accurate because the slugs are highly aerated and the h/D are smaller. This can sometimes make it difficult to differentiate, visually, slugs from large amplitude waves (see Fig. 11). In these cases measurements of the distinctive pressure pulses, characterizing slugs, can be used to differentiate fast moving slugs from slow moving large amplitude waves at the outlet of the pipe.

Average lengths measured close to the pipe outlet are not sensitive to changes in U_{SL} (from $U_{SL} = 0.6$ m/s to $U_{SL} = 1.2$ m/s) and weakly sensitive to U_{SG}. A value of $\bar{L}_S/D \cong 18$ was obtained for $U_{SG} = 2$–4 m/s. For $U_{SG} = 1$ m/s and for $U_{SG} > 4$ m/s slightly smaller lengths, $\bar{L}_S/D \cong 16$, were observed. Grenier et al. (1997) carried out a study in a 0.053 m pipe, which had a length of 90 m ($L/D = 1700$), at $U_{SG} < 2$ m/s and $U_{SL} = 0.35$–1.6 m/s. Provided the mixture velocity was greater than 1.2 m/s the mean slug length was found to be $\bar{L}_S/D = 20$. A similar result was obtained by Ferre (1979) and by Bernicott and Drouffe (1991) in a pipe with $L/D = 9500$. It is of interest that this roughly equals to the value of 18 found by Woods in a 0.0763 m pipe with a L/D that is over an order of magnitude smaller.

The experiments of Nydal et al. (1992) are of particular interest to this study since a large range of gas (0.5–20 m/s) and liquid (0.6–3.5 m/s) superficial velocities were considered. Two pipes with diameters 0.053 m and 0.09 m, and with a length of 17 m were used. They measured $\bar{L}_S/D = 15$–20 in the 0.053 m pipe and $\bar{L}_S/D = 12$–16 in the 0.09 m pipe. Experiments by Saether et al. (1990) in a 0.032 m pipe with $L/D = 560$ gave $\bar{L}_S/D = 27$ for $(U_{SG}+U_{SL}) = 7.58$ (m/s).

A striking feature of measurements such as shown in Fig. 6 (observed by many investigators) is the wide range of slug lengths. Woods (1998) obtained measurements of the standard deviation of slugs observed close to the outlet of a 0.0763 m pipe. He obtained a value of σ_{LS} of about 0.55 m for $U_{SG} = 2$–4 m/s, 0.4 m for $U_{SG} = 1$ m/s and 0.5 m for $U_{SG} > 4$ m/s. The ratio $\sigma_{LS}/\bar{L}_S$ was measured as 0.35 for $U_{GS} = 1$ m/s, 0.39 for $U_{GS} = 2$ m/s and 0.41 for $U_{SG} = 2.5$–7 m/s. Nydal et al. (1992) obtained $\sigma_{LS}/\bar{L}_S = 0.37$ (which is close to that measured by Woods) for $U_M = U_{SL} + U_{SG} \leqslant 5$ m/s. Nydal et al. measured a drop off of σ_{LS} at $U_M > 5$ m/s, that was not observed by Woods. A value of $\sigma_{LS}/\bar{L}_S$ of 0.31 was obtained at $U_M = 10$ m/s. The Saether et al. (1990) experiments at $U_{SL} + U_{SG} = 7.58$ m/s gave $\sigma_{LS}/\bar{L}_S = 0.26$.

A number of researchers have suggested that there is a minimum length below which slugs are unstable (Taitel et al., 1980). However, there have been differences in opinion as to its value (Barnea and Brauner, 1985; Dukler et al., 1985). Our measurements of $\bar{L}_S$ and dL_S/dt give $(\bar{L}_S/D)$ min $\cong 5$. The cumulative probability density function of slug lengths obtained at $U_{SG} = 5.0$ m/s by Nydal et al. (1992) gives (L_S/D)min $\cong 8$. Measurements by Grenier et al. (1997) at $U_{SG} = 0.5$ m/s give a 100% chance for a slug with $L_S/D = 9$ to disappear.

The probability density function for slug length shows a paucity of slugs with low L_S/D and a long tail for large L_S/D (Fig. 7). This suggests that it could be described by a lognormal function. This has been verified by Brill et al. (1981) and by Nydal et al. (1992). Not surprisingly, our measurements are also represented by

$$p(L_S) = \frac{1}{L_S C_2 \sqrt{2\pi}} \exp\left[\frac{-(\ln L_S - C_1)^2}{2C_2^2}\right] \tag{15}$$

where constants C_1 and C_2 can be obtained from measurements of $\bar{L}_S$ and σ^2_{LS}.

5. Formation of slugs

As mentioned in Section 1, a principal focus of this paper is large liquid flows for which slugs form from waves generated near the inlet and for which a Froude number characterizing the liquid flow at the inlet is greater than unity. One goal is to provide information on how these disturbances evolve into the "fully developed" pattern described in Section 3, and thus develop theories that describe the frequency of slugging and the slug length probability function. If the distribution is lognormal the latter goal would involve the prediction of the average and the standard deviation of the slug lengths.

The behavior of large interfacial disturbances near the inlet was studied with conductance probes at $L/D = 2$ and 4 and at $L/D = 10$ and 12. These were used to determine the velocity of large amplitude disturbances. Measurements were made of the correlation coefficient defined as

$$\rho_{xy}(\tau) = \frac{E\lfloor h_x(t)h_y(t+\tau)\rfloor}{\sigma_x\sigma_y} - \frac{\mu_x\mu_y}{\sigma_x\sigma_y} \tag{16}$$

where h_x is the upstream wave height profile, h_y, the downstream profile σ_x, σ_y, the standard deviations, μ_x, μ_y the means. A pressure transducer at $L/D = 22$ reflected the passage of a stug. However, these pressure pulsations are not so pronounced as when the transducer is located at the end of the pipe since they are affected by slugs discharging from the pipe.

Figs. 9–11 give measurements at $U_{SL} = 1.0$ m/s and $U_{SG} = 1.8$, 3.0, 6.75 m/s. All show a highly disturbed interface at $L/D = 4.0$. The frequency of the disturbances decreases dramatically downstream until at $L/D = 200$ an approximately "fully developed" slug pattern is observed. Sharp drops in the level behind a disturbance which are the signature for the presence of incipient slugs are seen at $L/D = 4$ in Figs. 9 and 10. The decrease in frequency occurs because incipient slugs consume the wavy stratified flow in front of them. At $L/D = 37$ both slugs and large amplitude waves are present. The large amplitude waves are consumed by the slugs so the number of peaks decreases between $L/D = 37$ and $L/D = 200$. Fig. 12 presents cumulative frequency spectra very close to the entry, $L/D = 2$, for $U_{SL} = 1.0$ m/s. It is noted that cumulative spectra are the same for $U_{SG} = 1.2$, 1.8, 2.3, and 3 m/s. The spectra are also the same for $U_{SG} = 4.7$, 6, and 6.7 m/s. The higher frequencies observed at the higher U_{SG} could be reflecting a change in the mechanism for generating waves.

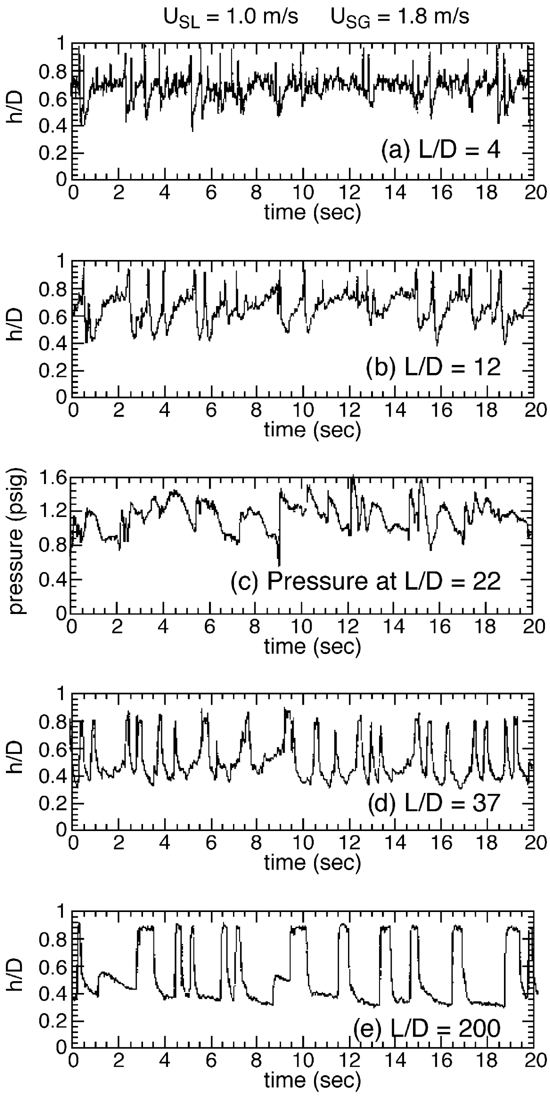

Fig. 9. Liquid holdup at five different locations and pressure pulsations at $L/D = 22$ for $U_{SG} = 1.8$ m/s, $U_{SL} = 1.0$ m/s. $D = 0.0763$ m.

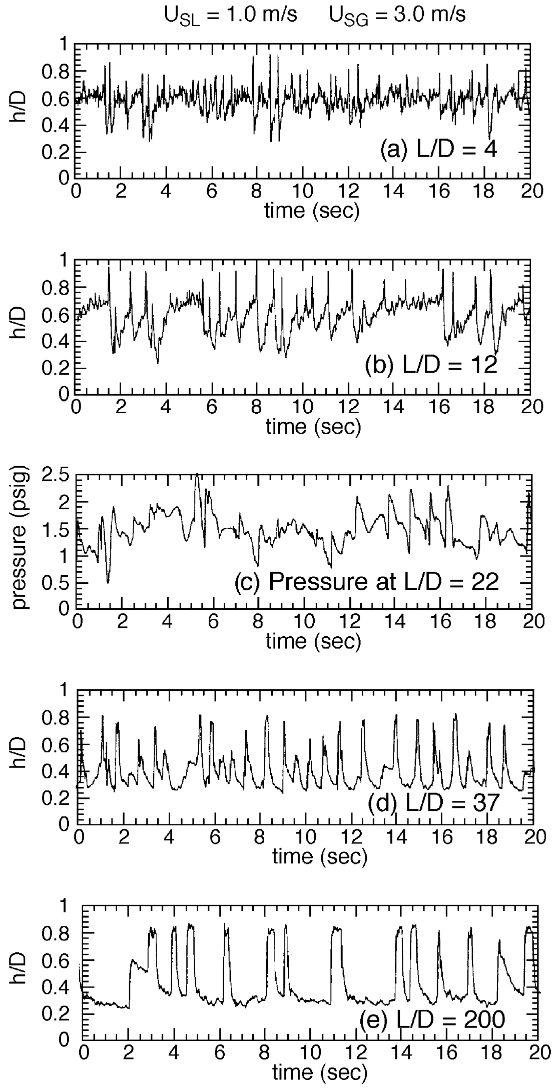

Fig. 10. Liquid holdup at five different locations and pressure pulsations at $L/D = 22$ for $U_{SG} = 3.0$ m/s, $U_{SL} = 1.0$ m/s, $D = 0.0763$.

The measured correlations for $U_{SG} = 1.8$ m/s, $U_{SL} = 1.0$ m/s, of the height fluctuations at $L/D = 12$ with the pressure fluctuations at $L/D = 22$ are presented in Fig. 13. A slug at $L/D = 12$ would be accompanied by a large positive pressure. When the slug reaches the pressure transducer a positive pressure increase would be recorded. From Eq. (8) a slug velocity of 3.4 m/s is calculated. Thus, the time for a slug to progress from $L/D = 12$ to $L/D = 22$ is 0.22 s. It is noted that the correlation presented Fig. 13 starts to assume positive values at time slightly larger than 0.22 s. The same behavior is observed for $U_{SG} = 1.0$ m/s. It was not observed for $U_{GS} = 6.75$ m/s. This suggests that slugs form at $L/D < 12$ for $U_{SG} = 1.0$, 1.8 m/s but not for $U_{SG} = 6.75$ m/s.

Measurements were made of the cross correlations of the liquid holdup at $L/D = 4$ and 12 and at 12 and 32. Peaks are associated with slugs and with disturbance waves. Velocities calculated from delay times at which the peaks were observed are plotted in Fig. 14 for $U_{SL} = 1.0$ m/s and different U_{SG}. The solid line represents the velocity of a slug, Eq. (8). The open circles are for disturbance wave velocities. The open squares are for slug velocities. For $U_{SG} < 3$ m/s one can conclude that the large amplitude waves at $L/D = 2$, 4, 10, 12 represent liquid that has bridged the pipe to form slugs. For $U_{SG} > 3$ m/s slugs are not seen to travel between $L/D = 4$ and 12, but they are seen to travel between $L/D = 12$ and $L/D = 37$. This indicates that, at high

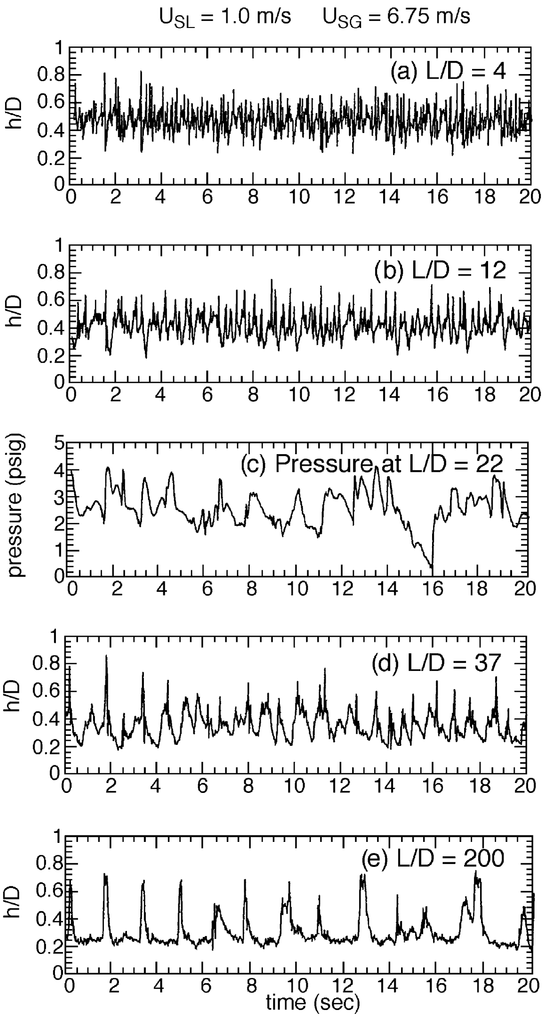

Fig. 11. Liquid holdup at five different locations and pressure pulsations at $L/D = 22$ for $U_{SG} = 6.75$ m/s, $U_{SL} = 1.0$ m/s, $D = 0.0763$ m.

gas velocities, the interfacial disturbances between $L/D = 4$ and $L/D = 12$ may, for the most part, be considered to be large amplitude coherent waves.

The measurements presented in Figs. 9–11 suggest the following picture of the behavior of interfacial disturbances generated close to the entrance. If $U_{SG} < 4$ m/s, incipient slugs develop very close to the entry. These grow if $h > h_0$ and decay if $h < h_0$. For h close to h_0 they will collapse if the slug length is not large enough. Collapsed slugs form large amplitude roll waves which propagate downstream. These are, eventually, accumulated by faster moving slugs. For $U_{SG} > 4$ m/s irregular large amplitude waves form close to the entrance. At a sufficient distance from the entrance two of these waves can coalesce to form a faster moving incipient slug (see Fig. 18 in WH). This slug will accumulate other waves as it propagates downstream. It will decay if it reaches a stratified flow for which the height is less than h_0.

The formation and decay of slugs at $U_{SG} < 4$ m/s described above is illustrated in Fig. 15 where the abscissa has been expanded by showing only 10 s. The flow conditions are $U_{SG} = 1.8$ m/s, $U_{SL} = 0.8$ m/s. A number of incipient slugs are identified at $L/D = 4.3$ by a high peak and a drop of the holdup to a value less than h_0 behind the disturbance. By $L/D = 21.5$ the tails of the slugs are, for the most part, overtaken by the stratified flow behind them, so the height at that location assumes a value of h_0. At $L/D = 110$ five slugs were identified from measurements of pressure pulsations at the end of the pipe, $L/D = 200$. Fig. 15 traces the previous

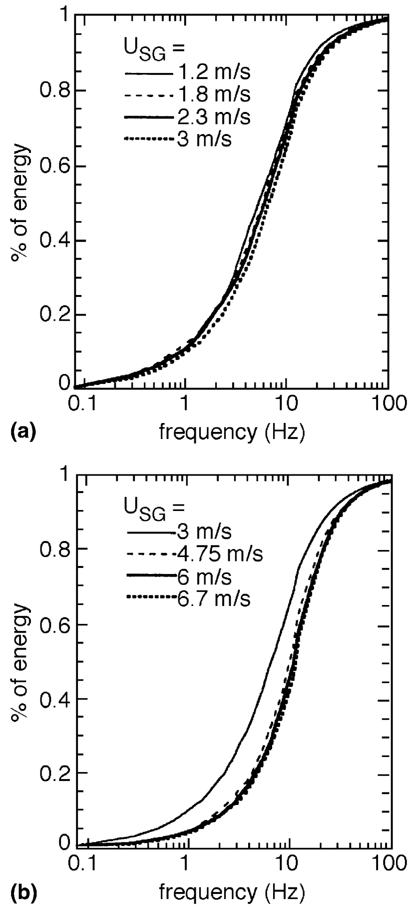

Fig. 12. Cumulative spectra of the fluctuations in h at $L/D = 2$ for $U_{SL} = 1.0$ m/s, $D = 0.0763$.

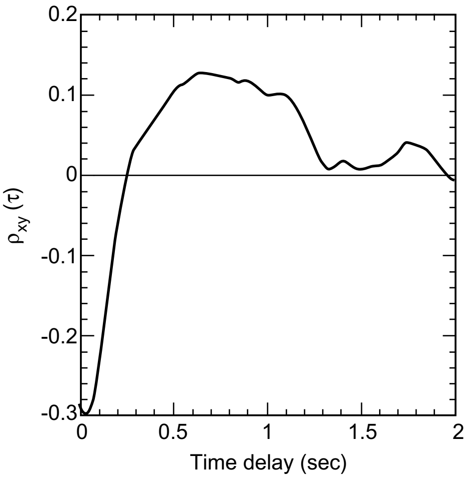

Fig. 13. Cross correlation between holdup fluctuations at $L/D = 12$ with the pressure fluctuations at $L/D = 22$ for $U_{SG} = 1.8$ m/s, $U_{SL} = 1.0$ m/s, $D = 0.0763$.

history of three slugs, S_1, S_2, S_3. It is noted that they increase in size between $L/D = 62$ and $L/D = 210$ by accumulating liquid in the roll waves or liquid in the stratified flow in front of them.

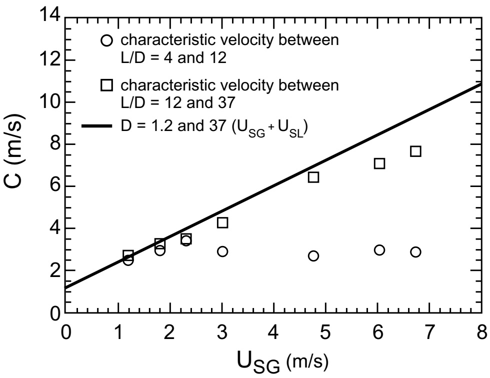

Fig. 14. Comparison of the velocities of disturbances between $L/D = 4$ and 12 and between $L/D = 12$ and 37 with the slug velocity, $D = 0.0763$ m.

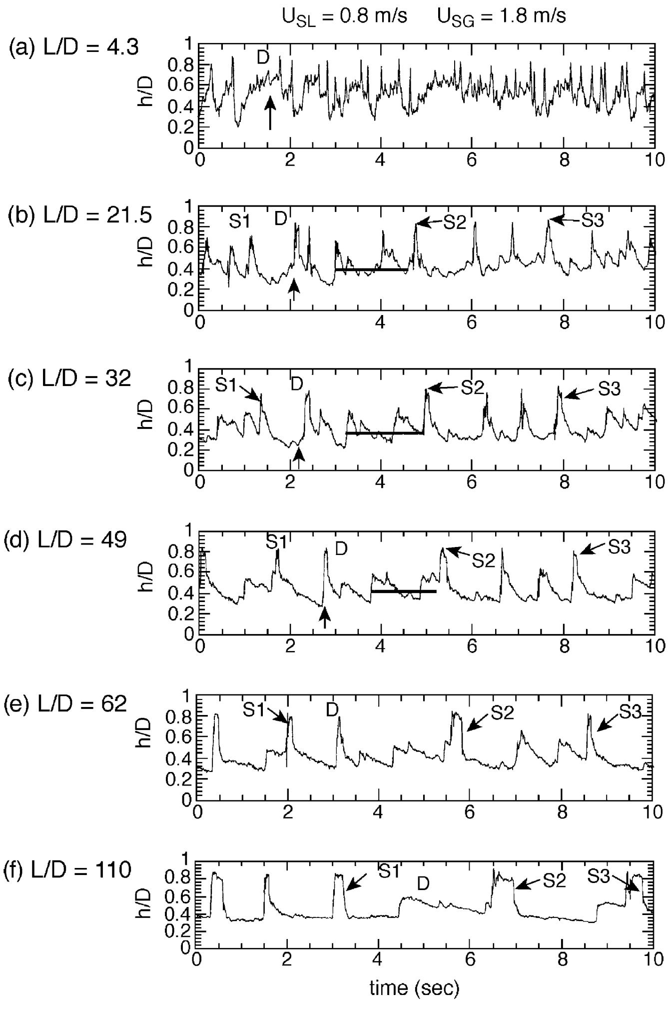

Fig. 15. Liquid holdup measurements at six different locations for $U_{SG} = 1.8$ m/s, $U_{SL} = 0.8$ m/s, $D = 0.0793$ m.

Large disturbance waves (incipient slugs) in front of S_2 are marked by a thick line at $L/D = 21.5$. These are consumed by the faster moving S_2, as shown in the tracings for $L/D = 21.5$, 32, 40, 62. The previous history of the roll wave at $L/D = 110$ is also shown. It is noted that it was an incipient slug at $L/D = 21.5$ that grew by consuming the liquid in front of it (marked by an arrow at $L/D = 4.3$). The height of the liquid stratified layer is less than h_0 at $L/D = 32$, 49, 62, so the incipient slug collapses between $L/D = 62$ and $L/D = 110$.

Video images at the disturbed interface close to the entry are shown in Fig. 16 for $U_{SG} = 1.8$ m/s, $U_{SL} = 1.0$ m/s (the conditions for Fig. 9). The liquid is dyed to present a contrast. The tee section in which the air and water are mixed is shown at the right. The probes at $L/D = 2$, 4 and 10 are shown in 16a. It should be noted that this is an expanded view of what is suggested in Fig. 9. The dimensionless length of $L/D = 10$ corresponds to a length of 0.76 m. The velocity of a slug at these conditions is 3.4 m/s so the photo in Fig. 16

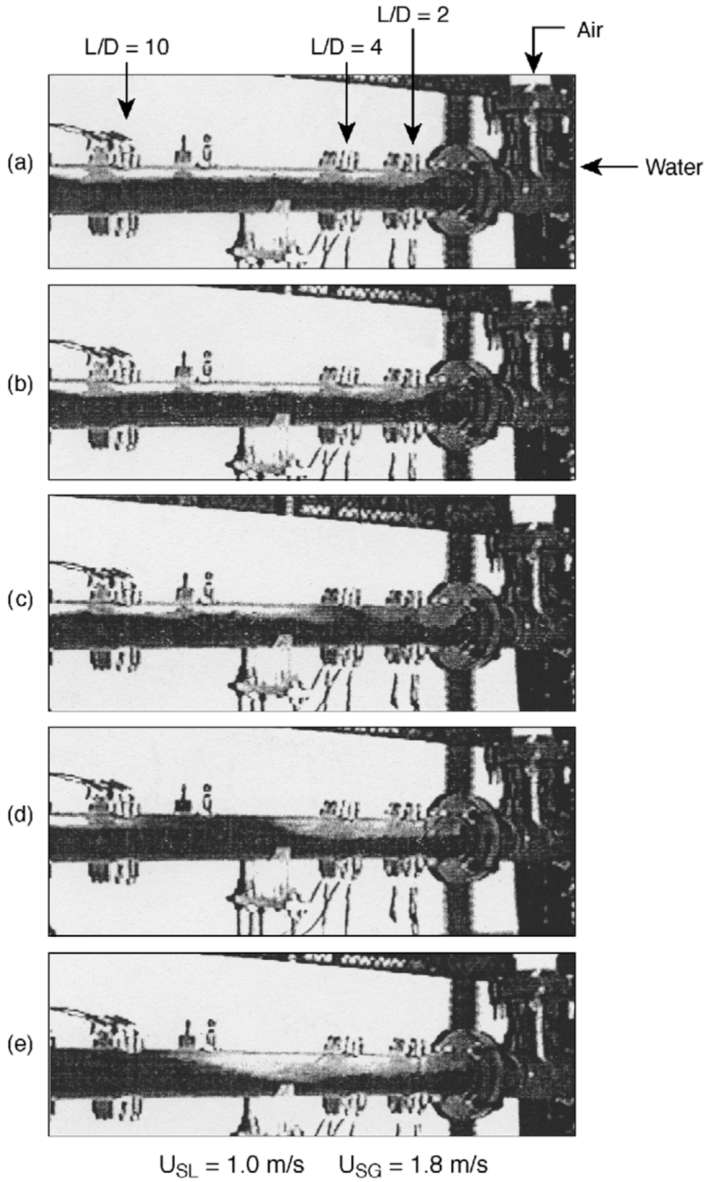

Fig. 16. Video images at the entrance for $U_{SG} = 1.8$ m/s and $U_{SL} = 1.0$ m/s, $D = 0.0763$ m.

would correspond to a time interval of 0.22 s in Fig. 9. The formation of an incipient slug is observed in 16a–c. It appears as a growing slug in 16d, where it has an $L_S/D = 5$. A developing tail is shown in Fig. 16d and e. The liquid in front of the slug is larger than h_0 so it is anticipated that this slug eventually reaches a large enough L_S/D to be stable. In 16e it can be seen that the depleted liquid is being replaced by an incoming bore.

Fig. 17 presents video pictures at the inlet for $U_{SG} = 3.0$ m/s, $U_{SL} = 1.0$ m/s. It is noted that the increase in gas velocity leads to a shallower stratified flow than shown in Fig. 16. A series of large amplitude waves are seen in 17a and b. By 17c a still larger wave is formed, probably as the result of wave coalescence. This grows so that it reaches the top wall and forms an incipient slug which moves out of view in 17e and f. A possible outcome is that this will not grow to a sufficient length to form a stable slug.

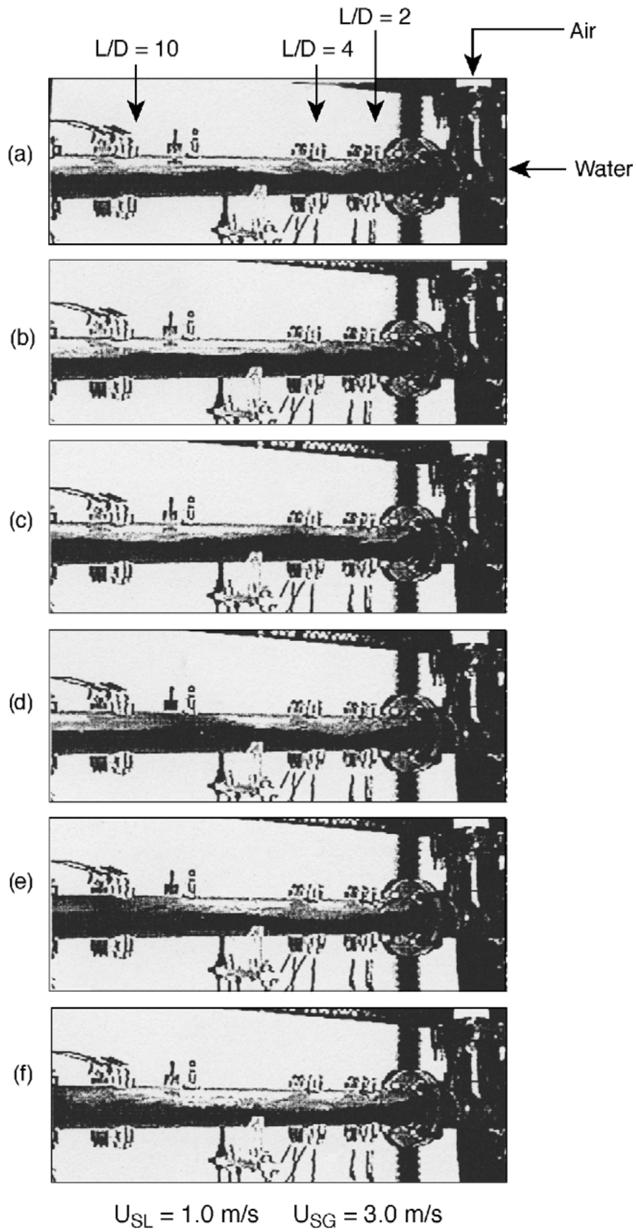

Fig. 17. Video images at the entrance for $U_{SG} = 3.0$ m/s and $U_{SL} = 1.0$ m/s, $D = 0.0763$ m.

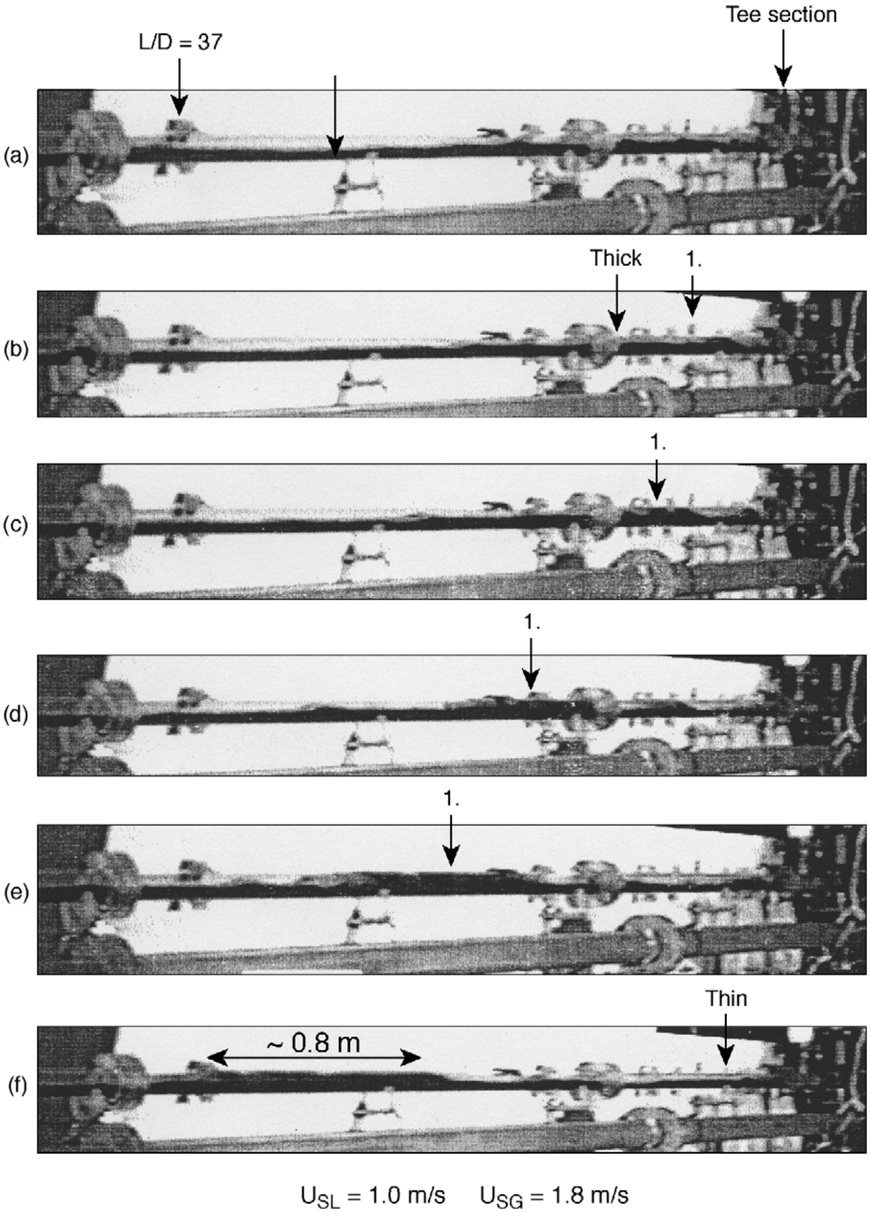

Fig. 18. Video images over a long length of pipe for $U_{SG} = 1.8$ m/s and $U_{SL} = 1.0$ m/s, $D = 0.0763$ m.

A sequence of video images is presented in Fig. 18 that illustrates the behavior of the liquid in the initial 3 m of the pipe for $U_{SG} = 1.8$ m/s and $U_{SL} = 1.0$ m/s. Fig. 18a gives the locations of the conductance probes. A stratified flow exists throughout the region shown in 18a. A disturbance, labeled as 1, is observed to bridge the pipe in 18b. The liquid layer in front of the incipient slug is thick enough to promote growth. The disturbance lengthens in c–f, and leaves a diminished layer in its rear. A trailing incipient slug observed in 18c and d, collapses and is not seen in 18e. The same is true for the incipient slug seen at the entrance in 18e. The length of slug 1 is approximately 0.8 m, so it is stable. Incipient slug 2 in 18g does not experience the rapid growth shown by slug 1 because it sees the diminished stratified flow in the wake of 1. Slug 2 does not reach a stable length in 18k and probably collapsed to create a thickened stratified flow that can be consumed by a trailing incipient slug.

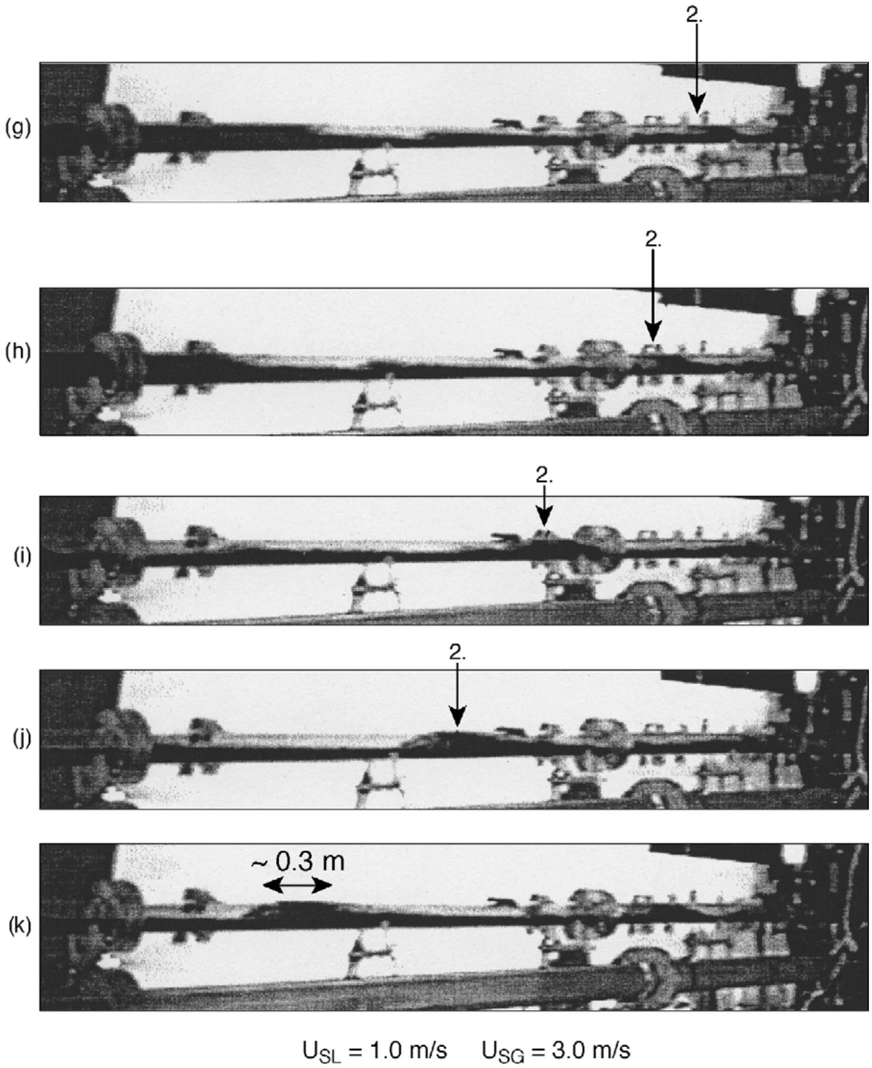

Fig. 18 (*continued*)

6. Concluding remarks

6.1. "Fully developed" slugs

This paper is related to a previous study of the formation slugs at low U_{SL} by Woods and Hanratty (1999). It extends this work to larger U_{SL} for which slugging occurs close to the pipe entrance.

The process for creating slugs at large U_{SL} is stochastic. The results are, therefore, expected to be qualitatively different from those obtained at low U_{SL} and low U_{SG}, for which slugging is regular.

A "fully developed" slug flow is defined for which stationary slugs with different lengths are immersed in a stratified flow whose height is the critical value needed to generate slugs, h_0.

Measurements of the frequency of slugging for a "fully developed" flow at large U_{SL} that are presented in this paper and in reports from other laboratories show strong effects of U_{SL}, U_{SG}, and pipe diameter. A plot of $f_S D/U_{SL}$ versus $U_{SL}/(U_{SG} + U_{SL})$ provides a rough first approximation. A number of authors have claimed that the variation of frequency with U_{SG} shows a minimum. This is supported by the extensive measurements by Fan in a horizontal 0.095 m pipe that are presented in this paper. The minimum is shown to occur for air–water flow at 4 m/s. An interpretation for this behavior is not available. We can only note that the superficial gas velocity at which this occurs for an air–water flow, 4 m/s, roughly defines the condition for which the generation of waves changes from a Jeffreys to a Kelvin Helmholtz mechanism. It should also be noted that the frequency of the waves close to the entry is higher at larger U_{SG}.

An examination of our measurements of holdup close to the outlet ($L/D = 200$) of a 0.0763 m pipe with an L/D ratio of 260 shows a "fully developed" slug flow at $U_{SG} = 1.05$ m/s and 1.8 m/s. At $U_{SG} = 2.4$ m/s the slugs were still accumulating mass from the stratified layer but the slug frequency is captured reasonably well.

A striking result from a consideration of our data and data from other laboratories covering a wide range of pipe lengths, L/D, is that the mean slug length in a "fully developed" pattern divided by the pipe diameter, $\bar{L}_S/D$, is relatively insensitive to changes in D, U_{SL}, U_{SG}. Values 12–27 have been reported, so a value of $\bar{L}_S/D = 20$ is representative. The spread of the data can be represented by the root-mean square of the deviations of L_S, designated by σ. Values of $\sigma/\bar{L}_S$ are also relatively insensitive to changes in flow variables. A value of 0.35 is representative of the data. The product of $\bar{L}_S/D = 20$ with $f_S D/U_{SL} = 0.05$ is in rough agreement with (9), indicating that the approach taken in deriving this equation is a valid one.

Presently available data suggest that the distribution of slug lengths is described reasonably well with a log normal function. The two constants that appear in this function can be evaluated if $\bar{L}_S/D$ and $\sigma/\bar{L}_S$ are known.

6.2. Development of a slug pattern

The theoretical understanding of the results outlined above need to be couched in a stochastic analysis, so the study of the development of slugs is an important aspect of this paper. The analysis in WH was carried out for situations in which slugs are formed far enough downstream that entry effects are unimportant. The point of the present paper is to look at cases for which the design of the entry could be important. Our measurements of slug frequency and of slug development does not reveal any obvious large effects of the entry.

A stochastic analysis should be similar to that carried out by WH at large gas velocities and small liquid velocities. One difference is that the pipe length needed to develop an unstable stratified flow (see Eq. (12)) moves closer to the inlet. For superficial gas velocities less than 3 m/s, L_D appears to be smaller than $4D$. For $U_{SG} \geqslant 3$ m/s, L_D/D appears to be between 4 and 12.

The observations presented, particularly the video images in Figs. 16–18, seem to contradict the simplifying assumption used by Woods that the height of the stratified flow immediately behind a slug is equal to h_0. The slugs will not grow if they encounter a layer with height h_0 and will decay if the slug length is smaller than some critical value, $(L_S/D)_{min}$. However, the video images show that slugs which are behind a recently formed slug may encounter a layer with a height less than h_0. This would contribute to the decay of the trailing slug. This behavior could be taken into account in the specification of $(L_S/D)_{min}$. Nevertheless, it might be interesting to explore a more sophisticated model of a slug which includes a developing tail.

Calculations by Woods (1998) for large U_{SL} are shown in Fig. 2. These assume that L_D in (12) is zero. It is noted that for a given $L(t)$ the largest slug that can be formed increases with decreasing L_D. The results presented in this paper suggest that, for the small values of U_{SG} represented by Fig. 2, a more appropriate selection of L_D/D would be a value of about 4. This would mean that largest value of L_S predicted by the theory could be too large by a value of 0.3 m if $L_D = 0$ is selected. This could explain the differences between experiment and theory shown in Fig. 2.

6.3. Petroleum pipelines

Results obtained in petroleum pipelines (Brill et al., 1981; Scott et al., 1987) show values of $\bar{L}_S/D$ which are an order of magnitude larger than what is observed in carefully controlled laboratory experiments. The notion that this occurs because C_B depends on slug length is not supported by experiments described in this paper. We can only speculate that the comparison reflects a difference in the way the slugs are formed in the pipeline. Many of the tests are conducted close to the conditions where the transition from a stratified to a slug flow occurs (Scott et al., 1987). Furthermore, one has to look carefully at the pipeline configuration before the entry to discover whether slugs are generated by a different mechanism from what was operative in the laboratory studies. For example, the Prudhoe bay test pipeline, used by Brill et al. (1981), had an entry section of 56 m that was declined at about 6.5°.

Acknowledgement

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